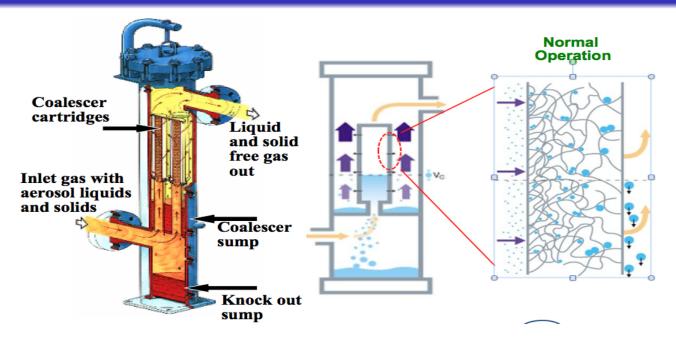
Saturation in Liquid/Gas Coalescence Presented by Mark Hurwitz, Pall Corporation

P. Aceves Sanchez, N. Bailey, M. Bruna, P. Cesana, B. Chakrabarti,
J. Chakraborty, C. J. Chapman, L. Cummings, M. Dallaston, J. Dietz, P. Dondl,
C. Finn, I. Griffiths, J. Herterich, C. Holloway, M. Hurwitz, A. Hutchinson,
R. Kgatle, D. Khoromskaia, A. Krupp, V. Lapin, N. Letchford, A. Lewis,
A. Manhart, M. Moore, A. Münch, H. Ockendon, J. Oliver, D. Paccagnan,
A. Patel, B. Piette, C. Please, R. Purvis, T. Rafferty, M. Saxton, D. Stuerzer,
Y. Sun, A. Tamsett, J. Uddin, T. Vo, Y. Wei, M. Zagórowska

ESGI100 University of Oxford

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The problem



If the concentration of the droplets on the fibres becomes too large, we have **saturation** and the gas flow may re-entrain the droplets.

We aim to model the processes occuring in the coalescer in order to understand saturation.

- Macro-scale model of the coalescer
- 2 Micro-scale considerations for droplets within the filter
- Constitutive relations for macro-model determined by micro-scale considerations

Macro-scale: Canonical one-dimensional problem

The quantities $\alpha_{lg}, \alpha_{lm}, \alpha_{g}, u_{g}, u_{l}, p$ vary with position and time according to

$$\alpha_{\rm lg} + \alpha_{\rm lm} + \alpha_{\rm g} = 1, \tag{1}$$

$$\frac{\partial}{\partial t}(\rho_{\rm g}\alpha_{\rm g}\phi) + \frac{\partial}{\partial x}(\rho_{\rm g}\alpha_{\rm g}u_{\rm g}\phi) = 0, \qquad (2)$$

$$\frac{\partial}{\partial t}(\rho_{\rm l}\alpha_{\rm lg}\phi) + \frac{\partial}{\partial x}(\rho_{\rm l}\alpha_{\rm lg}\phi u_{\rm g}) = -f_{\rm d},\tag{3}$$

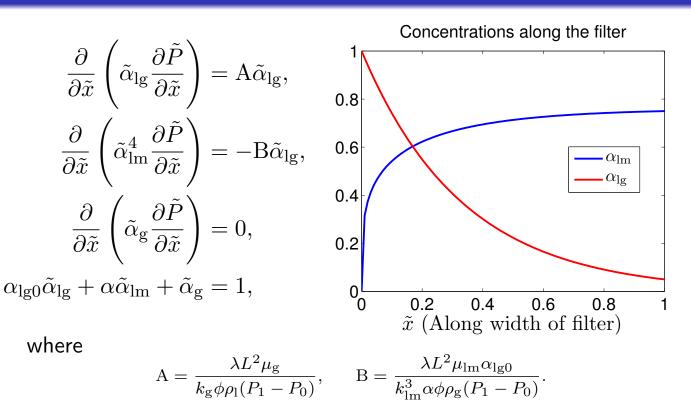
$$\frac{\partial}{\partial t}(\rho_l \alpha_{\rm lm} \phi) + \frac{\partial}{\partial x}(\rho_l \alpha_{\rm lm} \phi u_{\rm l}) = f_{\rm d}, \tag{4}$$

$$\frac{\partial}{\partial t} \left[(\rho_{\rm g} \alpha_{\rm lg} + \rho_{\rm g} \alpha_{\rm g}) \phi u_{\rm g} \right] = -\frac{\partial p}{\partial x} - \frac{\mu_{\rm g}}{k_{\rm g}} \phi u_{\rm g}, \tag{5}$$

$$\frac{\partial}{\partial t}(\rho_{\rm l}\alpha_{\rm lm}\phi u_{\rm l}) = -\frac{\partial p}{\partial x} - F_{\rm lm}.$$
 (6)

Here, $f_d = f_d(\alpha_{lg}, \alpha_{lm}, u_g)$ and $F_{lm} = F_{lm}(\alpha_{lm}, u_g, u_l, \lambda_{lm})$ are determined by micro-scale considerations.

Macro-scale: Simple case in 1D



We assumed that $\tilde{\alpha}_{\rm g} \approx 1 \Rightarrow P$ is linear.

 \blacktriangleright Analytical solutions for α_{lg} and α_{lm} .

Macro-scale: Simple case in 2D

$$\begin{split} & \frac{\partial}{\partial \tilde{x}} \left(\tilde{\alpha}_{\mathrm{lg}} \frac{\partial \tilde{P}}{\partial \tilde{x}} \right) + \epsilon^2 \frac{\partial}{\partial \tilde{y}} \left(\tilde{\alpha}_{\mathrm{lg}} \frac{\partial \tilde{P}}{\partial \tilde{y}} \right) = \mathrm{A} \tilde{\alpha}_{\mathrm{lg}}, \\ & - \left[\frac{\partial}{\partial \tilde{x}} \left(\tilde{\alpha}_{\mathrm{lm}}^4 \frac{\partial \tilde{P}}{\partial \tilde{x}} \right) + \epsilon^2 \frac{\partial}{\partial \tilde{y}} \left(\tilde{\alpha}_{\mathrm{lm}}^4 \frac{\partial \tilde{P}}{\partial \tilde{y}} \right) \right] - \mathrm{E} \frac{\partial}{\partial \tilde{y}} (\tilde{\alpha}_{\mathrm{lm}}) = \mathrm{B} \tilde{\alpha}_{\mathrm{lg}}, \\ & \frac{\partial}{\partial \tilde{x}} \left(\tilde{\alpha}_{\mathrm{g}} \frac{\partial \tilde{P}}{\partial \tilde{x}} \right) + \epsilon^2 \frac{\partial}{\partial \tilde{y}} \left(\tilde{\alpha}_{\mathrm{g}} \frac{\partial \tilde{P}}{\partial \tilde{y}} \right) = 0, \\ & \frac{\partial}{\partial \tilde{x}} \left(\tilde{\alpha}_{\mathrm{g}} \frac{\partial \tilde{P}}{\partial \tilde{x}} \right) + \epsilon^2 \frac{\partial}{\partial \tilde{y}} \left(\tilde{\alpha}_{\mathrm{g}} \frac{\partial \tilde{P}}{\partial \tilde{y}} \right) = 0, \\ & \text{Contour plot of } \alpha_{\mathrm{lm}} \\ & \mathrm{E} = \frac{\epsilon \mu_{\mathrm{lm}} L \rho_{\mathrm{lg}}}{k_{\mathrm{lm}}^3 (P_1 - P_O)}. \end{split}$$

W



Micro-scale considerations I

• Capture rate of droplets:

$$f_{\rm d} = \alpha_{\rm lg} \rho_{\rm l} v_{\rm g} (1 - \phi) (d_{\rm f} + d_{\rm d}) \tag{7}$$

Penetration length $\lambda = f_{
m d}/v_{
m g} pprox 0.2$ mm

- Stokes flow along fibres
 - Assume fibres coated by liquid
 - Stokes flow of liquid along the fibre
 - R_0 : fibre radius, $R_1 = R_0 + h$ radius of coated fibre Flow:

$$Q = \frac{2\pi}{\mu} \frac{\partial p}{\partial x} \left[\frac{(R_1^2 - R_0^2)(3R_1^2 - R_0^2)}{16} - \frac{R_1^4}{4} \ln\left(\frac{R_1}{R_0}\right) \right]$$
(8)

Experimental data $\rightarrow h \approx 0.1 \mu m$. Coated fibres can sustain large fluid flow.

Micro-scale considerations II

• Droplet displacement

Balance surface tension variation when moving on fibre against drag force on droplet:

$$r_{\rm d} = \frac{(N_{\rm f} - 1)\gamma d_{\rm f}}{3\eta u_{\rm g}} \tag{9}$$

Droplets with radii below $r_{\rm d}$ do not move.

• Continuum equation to implement this:

$$\alpha_{\rm lm} = n\lambda$$

$$\frac{\partial \alpha_{\rm lm}}{\partial t} + \frac{\partial u_{\rm l} \alpha_{\rm lm}}{\partial x} = u_{\rm g} \alpha_{\rm lg} (1 - \phi)$$

$$u_{\rm l} = \begin{cases} 0, & V_{\rm drop} < V_{\rm drop \ minimum} \\ u_{\rm g} + \frac{\partial p}{\partial x} \frac{1}{3\pi\mu r_{\rm d}\eta}, & V_{\rm drop} > V_{\rm drop \ minimum} \end{cases}$$
(10)

- One-dimensional lattice model that includes crossings and open space
- Particles move freely in the open space and as a function of size on the crossings

- Two regimes; $r(k) = \begin{cases} e^k & k \text{ is small} \\ \text{Crossover behaviour} \\ k^{-3} & k \text{ is large} \end{cases}$
- Saturation transition dependent on parameters such as crossover and influx of particles
- Do we provide a tool to help design a coalescent? Find optimal operating conditions?

The mathematics; Markov generator

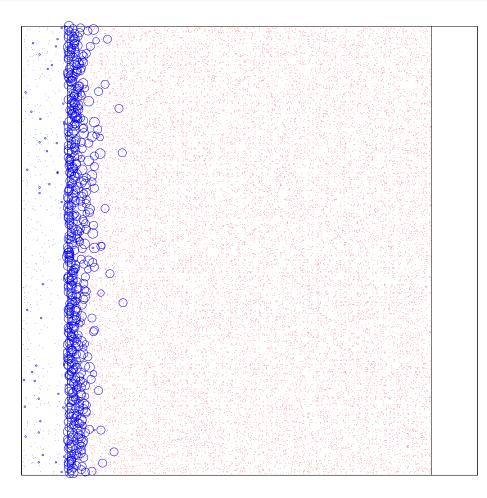
$$\mathcal{L}f(\eta) = \sum r(\eta_x)(f(\eta') - f(\eta)) \quad \text{Paticles on crossings} \\ + \sum u(f(\eta') - f(\eta)) \quad \text{Term due to particles in open space} \\ + u(f(\eta + \delta_1) - f(\eta)) \quad \text{Term due to incoming particles} \end{cases}$$

- Defining saturation When a single particle is of N (the total number of particles)
- Controlling the average particle number $\frac{d}{dt}\mathbb{E}(N(t)) = u \mathbb{E}(\eta_L r(\eta_L))$

Markov approach: working coalescer

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Markov approach: saturated coalescer



- Multiple macro-scale models to describe the physical processes going on in the coalescer:
 - Continuum model
 - Markov approach
- Two possible micro-scale mechanisms for flow of liquid droplets through the fibre network:
 - Stick/slip idea
 - Stick/film-flow idea
- Simulations for continuum and Markov models