Saturation in Liquid/Gas Coalescence
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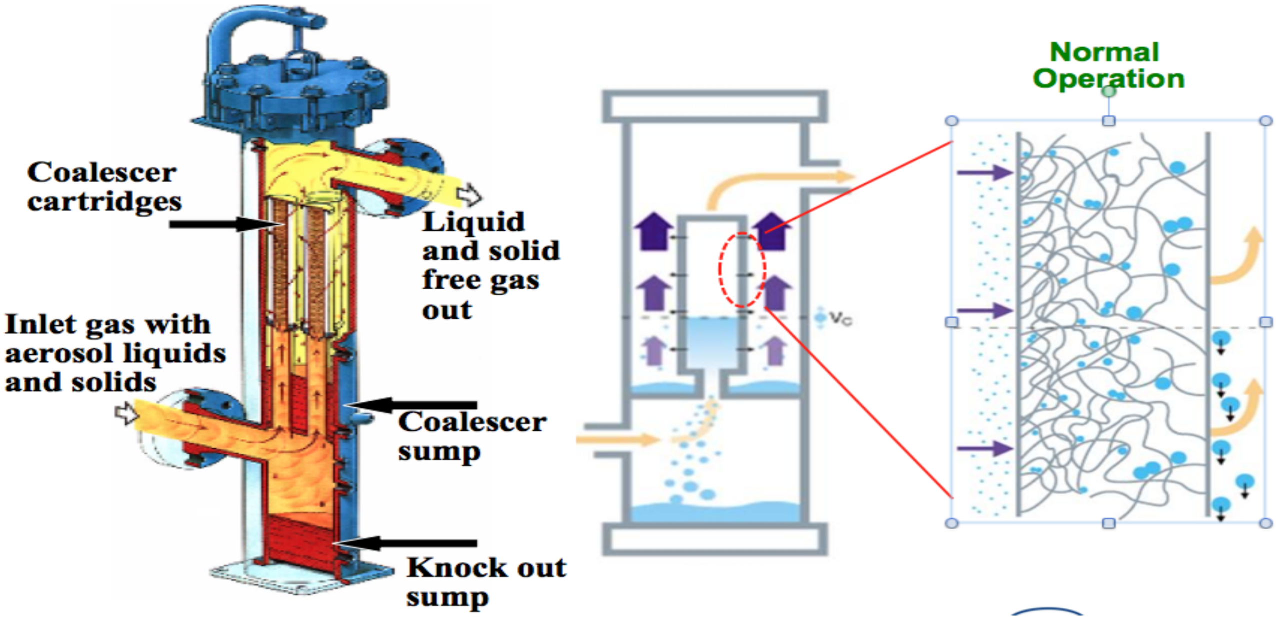
The problem



If the concentration of the droplets on the fibres becomes too large, we have **saturation** and the gas flow may re-entrain the droplets.

We aim to model the processes occuring in the coalescer in order to understand saturation.

Our approach

1. Macro-scale model of the coalescer
2. Micro-scale considerations for droplets within the filter
3. Constitutive relations for macro-model determined by micro-scale considerations

Macro-scale: Canonical one-dimensional problem

The quantities $\alpha_{\mathrm{lg}}, \alpha_{\mathrm{lm}}, \alpha_{\mathrm{g}}, u_{\mathrm{g}}, u_{\mathrm{l}}, p$ vary with position and time according to

$$\alpha_{\mathrm{lg}} + \alpha_{\mathrm{lm}} + \alpha_{\mathrm{g}} = 1, \tag{1}$$

$$\frac{\partial}{\partial t}(\rho_{\mathrm{g}}\alpha_{\mathrm{g}}\phi) + \frac{\partial}{\partial x}(\rho_{\mathrm{g}}\alpha_{\mathrm{g}}u_{\mathrm{g}}\phi) = 0, \tag{2}$$

$$\frac{\partial}{\partial t}(\rho_{\mathrm{l}}\alpha_{\mathrm{lg}}\phi) + \frac{\partial}{\partial x}(\rho_{\mathrm{l}}\alpha_{\mathrm{lg}}\phi u_{\mathrm{g}}) = -f_{\mathrm{d}}, \tag{3}$$

$$\frac{\partial}{\partial t}(\rho_{l}\alpha_{\mathrm{lm}}\phi) + \frac{\partial}{\partial x}(\rho_{\mathrm{l}}\alpha_{\mathrm{lm}}\phi u_{\mathrm{l}}) = f_{\mathrm{d}}, \tag{4}$$

$$\frac{\partial}{\partial t}\left[(\rho_{\mathrm{g}}\alpha_{\mathrm{lg}} + \rho_{\mathrm{g}}\alpha_{\mathrm{g}})\phi u_{\mathrm{g}}\right] = -\frac{\partial p}{\partial x} - \frac{\mu_{\mathrm{g}}}{k_{\mathrm{g}}}\phi u_{\mathrm{g}}, \tag{5}$$

$$\frac{\partial}{\partial t}(\rho_{\mathrm{l}}\alpha_{\mathrm{lm}}\phi u_{\mathrm{l}}) = -\frac{\partial p}{\partial x} - F_{\mathrm{lm}}. \tag{6}$$

Here, $f_{\mathrm{d}} = f_{\mathrm{d}}(\alpha_{\mathrm{lg}}, \alpha_{\mathrm{lm}}, u_{\mathrm{g}})$ and $F_{\mathrm{lm}} = F_{\mathrm{lm}}(\alpha_{\mathrm{lm}}, u_{\mathrm{g}}, u_{\mathrm{l}}, \lambda_{\mathrm{lm}})$ are determined by micro-scale considerations.

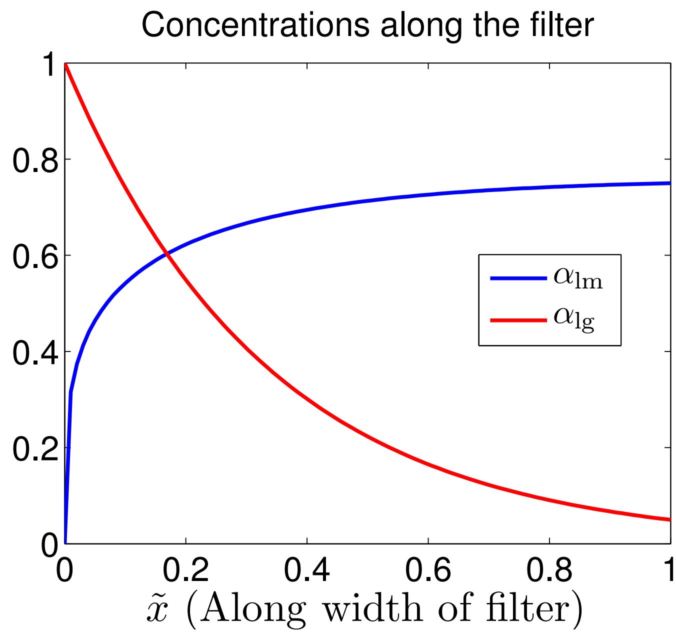
Macro-scale: Simple case in 1D

$$\frac{\partial}{\partial \tilde{x}}\left(\tilde{\alpha}_{\mathrm{lg}}\frac{\partial \tilde{P}}{\partial \tilde{x}}\right) = \mathrm{A}\tilde{\alpha}_{\mathrm{lg}},$$

$$\frac{\partial}{\partial \tilde{x}}\left(\tilde{\alpha}_{\mathrm{lm}}^{4}\frac{\partial \tilde{P}}{\partial \tilde{x}}\right) = -\mathrm{B}\tilde{\alpha}_{\mathrm{lg}},$$

$$\frac{\partial}{\partial \tilde{x}}\left(\tilde{\alpha}_{\mathrm{g}}\frac{\partial \tilde{P}}{\partial \tilde{x}}\right) = 0,$$

$$\alpha_{\mathrm{lg}0}\tilde{\alpha}_{\mathrm{lg}} + \alpha\tilde{\alpha}_{\mathrm{lm}} + \tilde{\alpha}_{\mathrm{g}} = 1,$$



where

$$\mathrm{A} = \frac{\lambda L^2 \mu_{\mathrm{g}}}{k_{\mathrm{g}}\phi\rho_{\mathrm{l}}(P_1 - P_0)}, \qquad \mathrm{B} = \frac{\lambda L^2 \mu_{\mathrm{lm}}\alpha_{\mathrm{lg}0}}{k_{\mathrm{lm}}^3 \alpha\phi\rho_{\mathrm{g}}(P_1 - P_0)}.$$

We assumed that $\tilde{\alpha}_{\mathrm{g}} \approx 1 \Rightarrow P$ is linear.

➥ Analytical solutions for α_{lg} and α_{lm}.

Macro-scale: Simple case in 2D

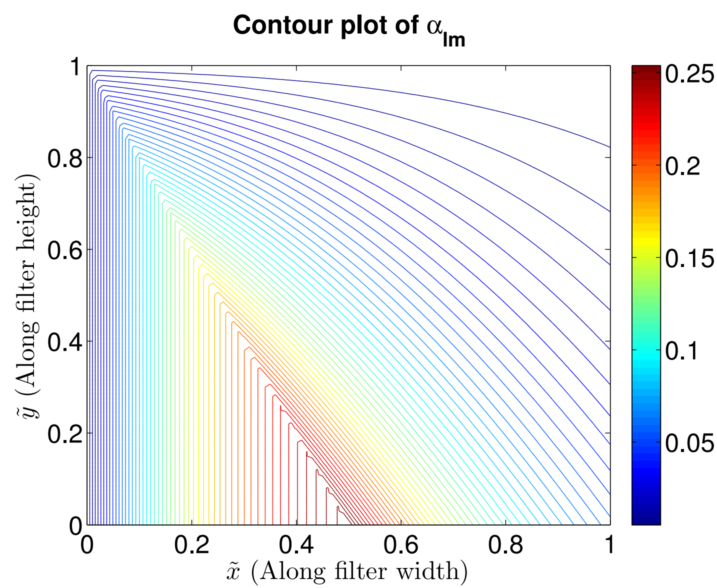
$$\frac{\partial}{\partial \tilde{x}}\left(\tilde{\alpha}_{\mathrm{lg}}\frac{\partial \tilde{P}}{\partial \tilde{x}}\right)+\epsilon^2\frac{\partial}{\partial \tilde{y}}\left(\tilde{\alpha}_{\mathrm{lg}}\frac{\partial \tilde{P}}{\partial \tilde{y}}\right)=\mathrm{A}\tilde{\alpha}_{\mathrm{lg}},$$

$$-\left[\frac{\partial}{\partial \tilde{x}}\left(\tilde{\alpha}_{\mathrm{lm}}^4\frac{\partial \tilde{P}}{\partial \tilde{x}}\right)+\epsilon^2\frac{\partial}{\partial \tilde{y}}\left(\tilde{\alpha}_{\mathrm{lm}}^4\frac{\partial \tilde{P}}{\partial \tilde{y}}\right)\right]-\mathrm{E}\frac{\partial}{\partial \tilde{\mathrm{y}}}(\tilde{\alpha}_{\mathrm{lm}})=\mathrm{B}\tilde{\alpha}_{\mathrm{lg}},$$

$$\frac{\partial}{\partial \tilde{x}}\left(\tilde{\alpha}_{\mathrm{g}}\frac{\partial \tilde{P}}{\partial \tilde{x}}\right)+\epsilon^2\frac{\partial}{\partial \tilde{y}}\left(\tilde{\alpha}_{\mathrm{g}}\frac{\partial \tilde{P}}{\partial \tilde{y}}\right)=0,$$

where

$$\mathrm{E}=\frac{\epsilon\mu_{\mathrm{lm}}L\rho_{\mathrm{lg}}}{k_{\mathrm{lm}}^3(P_1-P_O)}.$$



Micro-scale considerations I

- Capture rate of droplets:

$$f_{\mathrm{d}} = \alpha_{\mathrm{lg}} \rho_{\mathrm{l}} v_{\mathrm{g}} (1 - \phi)(d_{\mathrm{f}} + d_{\mathrm{d}}) \tag{7}$$

  Penetration length $\lambda = f_{\mathrm{d}} / v_{\mathrm{g}} \approx 0.2$ mm
- Stokes flow along fibres
  - Assume fibres coated by liquid
  - Stokes flow of liquid along the fibre
  - R_0 : fibre radius, $R_1 = R_0 + h$ radius of coated fibre

  Flow:

$$Q = \frac{2\pi}{\mu} \frac{\partial p}{\partial x} \left[\frac{(R_1^2 - R_0^2)(3R_1^2 - R_0^2)}{16} - \frac{R_1^4}{4} \ln\left(\frac{R_1}{R_0}\right) \right] \tag{8}$$

  Experimental data $\rightarrow h \approx 0.1 \mu m$.
  Coated fibres can sustain large fluid flow.

Micro-scale considerations II

- Droplet displacement
  Balance surface tension variation when moving on fibre against drag force on droplet:

$$r_{\mathrm{d}} = \frac{(N_{\mathrm{f}} - 1)\gamma d_{\mathrm{f}}}{3\eta u_{\mathrm{g}}} \tag{9}$$

  Droplets with radii below r_{d} do not move.
- Continuum equation to implement this:

$$\begin{aligned}
\alpha_{\mathrm{lm}} &= n\lambda \\
\frac{\partial \alpha_{\mathrm{lm}}}{\partial t} + \frac{\partial u_{\mathrm{l}} \alpha_{\mathrm{lm}}}{\partial x} &= u_{\mathrm{g}} \alpha_{\mathrm{lg}} (1 - \phi) \qquad (10) \\
u_{\mathrm{l}} &= \begin{cases} 0, & V_{\mathrm{drop}} < V_{\mathrm{drop\ minimum}} \\ u_{\mathrm{g}} + \frac{\partial p}{\partial x} \frac{1}{3\pi\mu r_{\mathrm{d}} \eta}, & V_{\mathrm{drop}} > V_{\mathrm{drop\ minimum}} \end{cases}
\end{aligned}$$

Markov approach I

- One-dimensional lattice model that includes crossings and open space
- Particles move freely in the open space and as a function of size on the crossings
- Two regimes; $r(k) = \begin{cases} e^k & k \text{ is small} \\ \text{Crossover behaviour} \\ k^{-3} & k \text{ is large} \end{cases}$
- Saturation transition dependent on parameters such as crossover and influx of particles
- Do we provide a tool to help design a coalescent? Find optimal operating conditions?

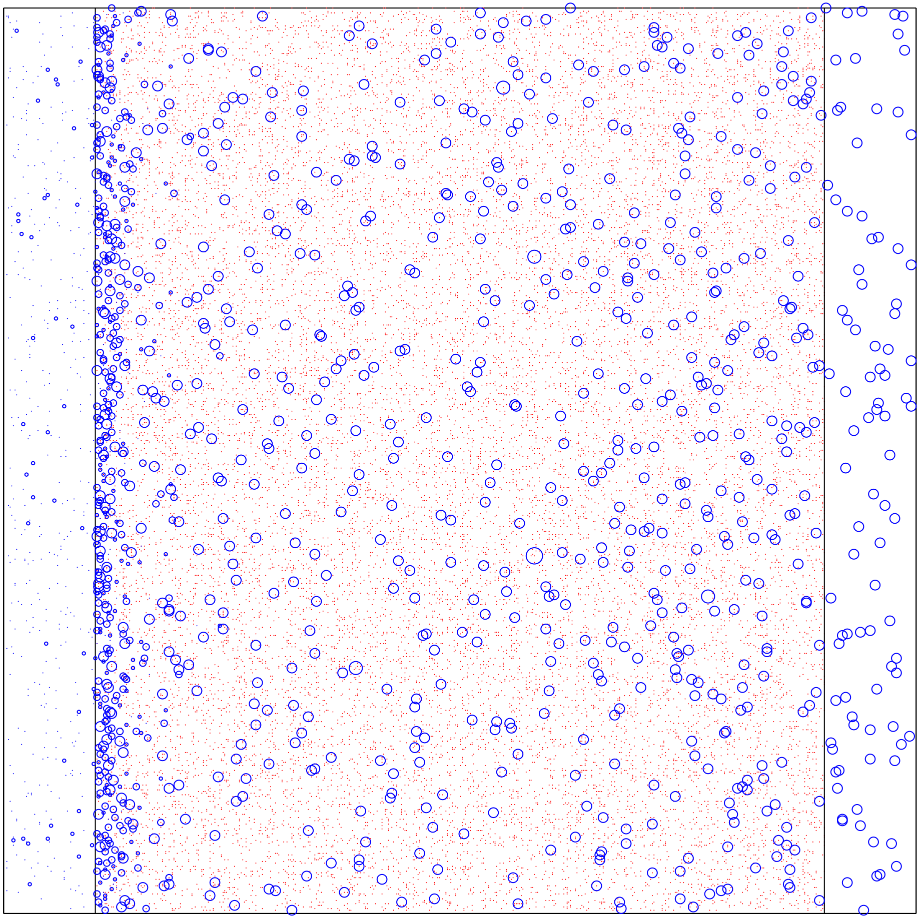
Markov approach II

The mathematics; Markov generator

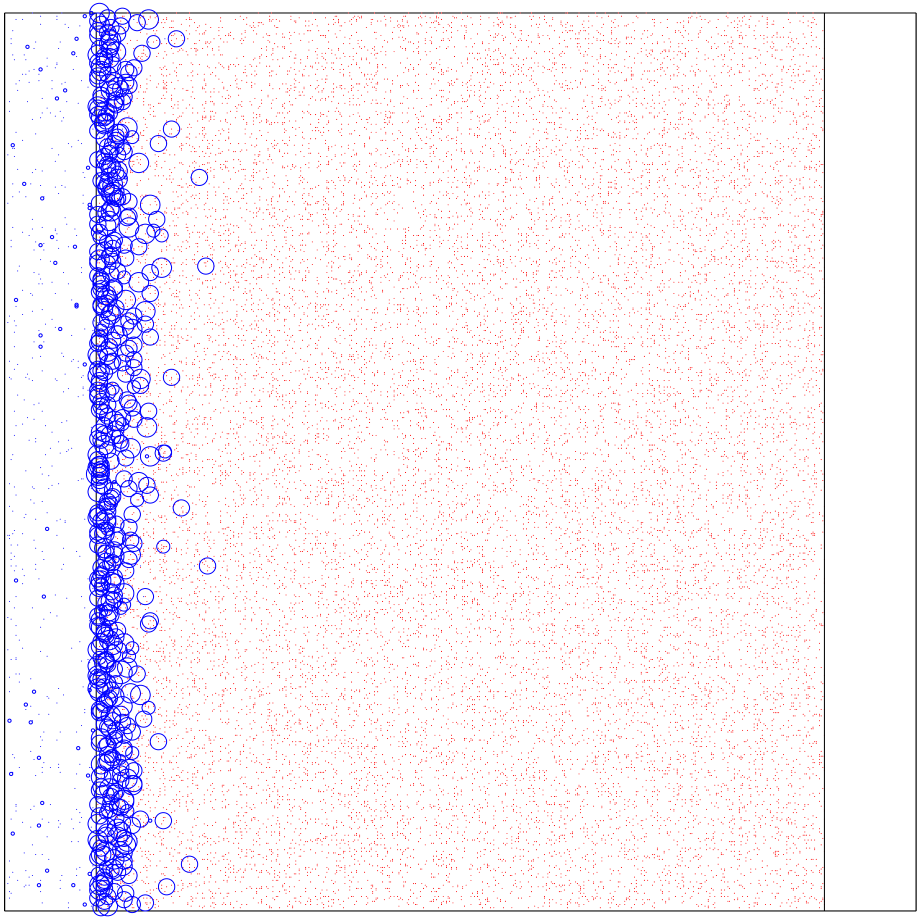
$$
\begin{aligned}
\mathcal{L}f(\eta) &= \sum r(\eta_x)(f(\eta') - f(\eta)) \quad \text{Paticles on crossings} \\
&+ \sum u(f(\eta') - f(\eta)) \quad \text{Term due to particles in open space} \\
&+ u(f(\eta + \delta_1) - f(\eta)) \quad \text{Term due to incoming particles}
\end{aligned}
$$

- Defining saturation - When a single particle is of N (the total number of particles)
- Controlling the average particle number - $\frac{d}{dt}\mathbb{E}(N(t)) = u - \mathbb{E}(\eta_L\, r(\eta_L))$

Markov approach: working coalescer



Markov approach: saturated coalescer



Results

- Multiple macro-scale models to describe the physical processes going on in the coalescer:
    - Continuum model
    - Markov approach
- Two possible micro-scale mechanisms for flow of liquid droplets through the fibre network:
    - Stick/slip idea
    - Stick/film-flow idea
- Simulations for continuum and Markov models